AC FUNDAMENTALS

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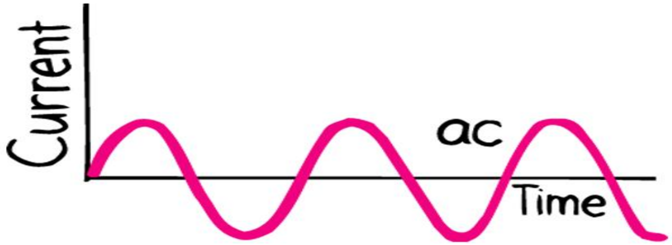
What is A.C. and D.C.

- In Alternating current, movement of electric charge periodically reverses its direction.



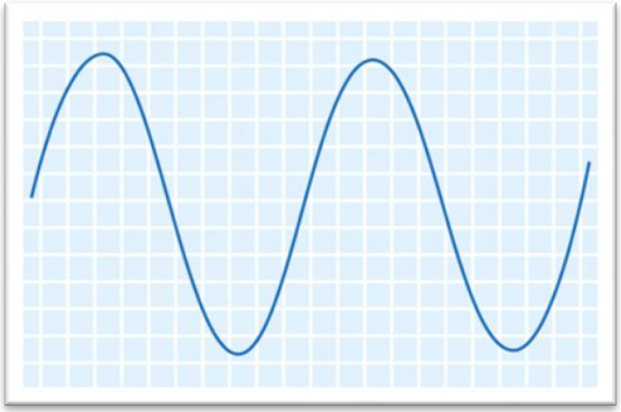
- Whereas in DC , flow of electric charge is only in one direction.

Why we use A.C. in homes

- AC voltage is capable of converting voltage levels just with use of transformers.
  - To transmit AC over a long distance, voltage is stepped up to 400 KV at generating stations and stepped down at a low level , 400/230 V for household and commercial utilization.
- AC motors are simple in construction, more efficient and robust as compared to DC motors

Sinusoidal Alternating Quantity

Alternating quantity that varies according to sin of angle θ.



The instantaneous value of a sine-wave voltage for any angle of rotation is expressed in the formula:

$V = V_m \sin\theta$

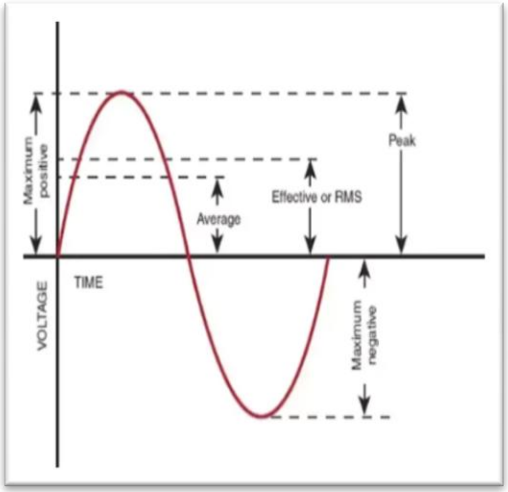
θ is the angle

V_m = the maximum voltage value

V = the instantaneous value of voltage at angle θ.

Terms to know

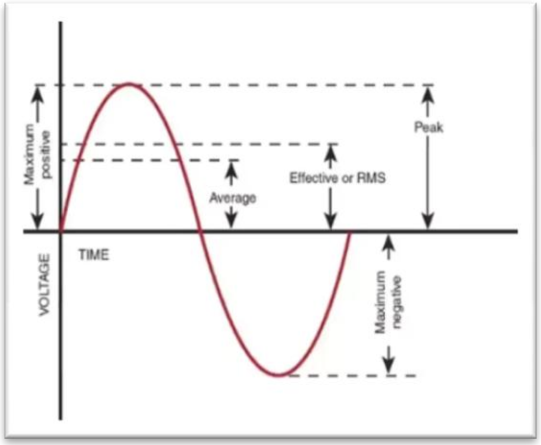
- Cycle
  - When an Alternating qty goes through complete set of positive and negative values
  - Goes through 360 electrical degrees.
- Alternation
  - One half cycle
- Time Period
  - Time taken to complete one cycle by AC.
- Frequency
  - No of cycle made per second.
- Amplitude
  - Maximum value attained by an alternating quantity in one cycle
  - Also called Peak Value / Max. Value



Expressing Alternating Quantaties

- In AC system, alternating current and voltage varies from instant to instant.

- Three ways to express
  - Peak Value
  - Average Value
  - RMS value



Average Value

- Average value of whole sinusoidal waveform over one complete cycle is zero as two halves cancel each other out.
  - So average value is taken over half a cycle and taken as 0.637 times peak value.

The alternating current varying sinusoidally is given by

I = I_m sin θ

Consider an elementary strip of thickness dθ in the + ive half cycle, i be its mid ordinates.

Thus, Area of strip = i dθ

Area of half cycle = $\int_0^{\pi} i d\theta$ = $\int_0^{\pi} I_m$ sin θ dθ

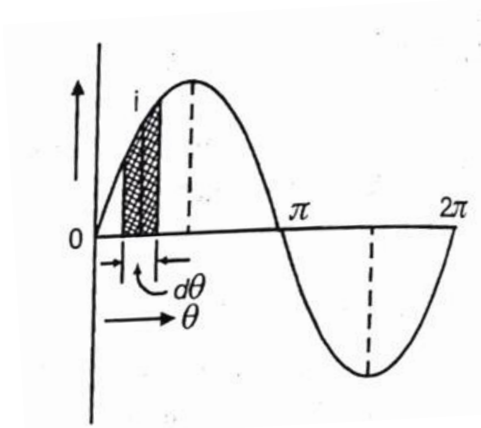
$= I_m \, (-cos\theta)_0^{\pi}$

= I_m {-(cosπ-cos0)}

= $2I_m$

Average Value I_{av} = $\frac{Area\ of\ alternation}{Base}$

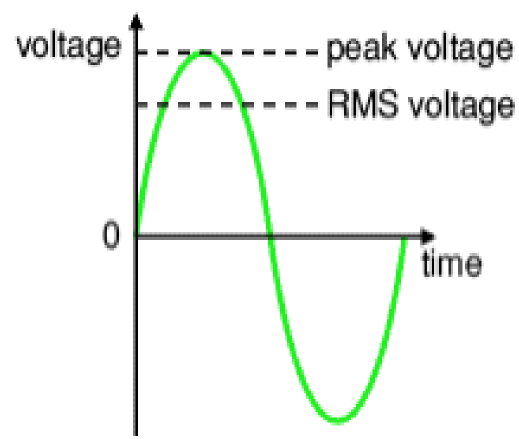
= $\frac{2I_m}{\pi}$ = 0.637 I_m



R.M.S. Value

- Root mean square value is effective value of varying voltage or current.
  - It is equivalent steady DC value that gives the same effect.

- Two methods of finding RMS value
  - Graphical Method
  - Analytical Method



R.M.S. Value

An alternating current is given by

$I = I_m \sin\theta$

Area of strip = $i^2 d\theta$

Area of squared wave in first half cycle

$\int_0^{\pi} i^2 d\theta = \int_0^{\pi} (I_m \sin\theta)^2 d\theta$

$= I_m^2 \int_0^{\pi} \sin^2\theta \cdot d\theta = I_m^2 \int_0^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$

$= \frac{1}{2} I_m^2 \int_0^{\pi} (1 - \cos 2\theta) \cdot d\theta = \frac{1}{2} I_m^2 \left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}$

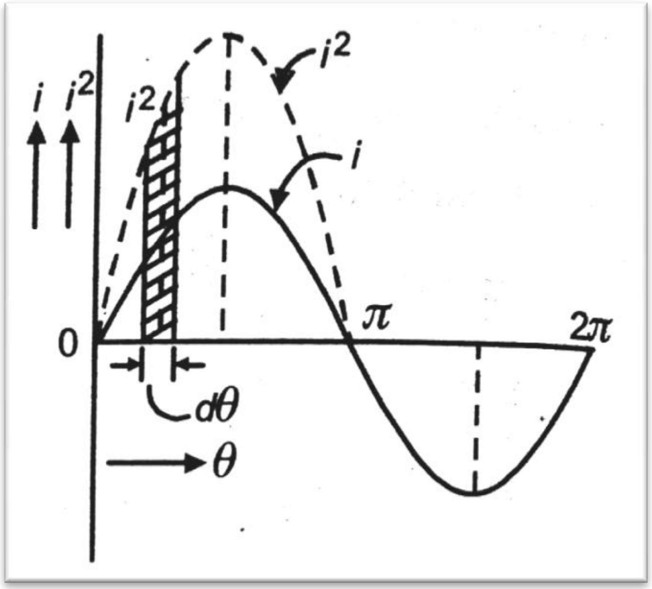
$= \frac{1}{2} I_m^2 \{(\pi - 0) - \frac{\sin 2\pi - \sin\theta}{2}\}$

$= \frac{1}{2} I_m^2 \{(\pi - 0) - (0 - 0)\}$

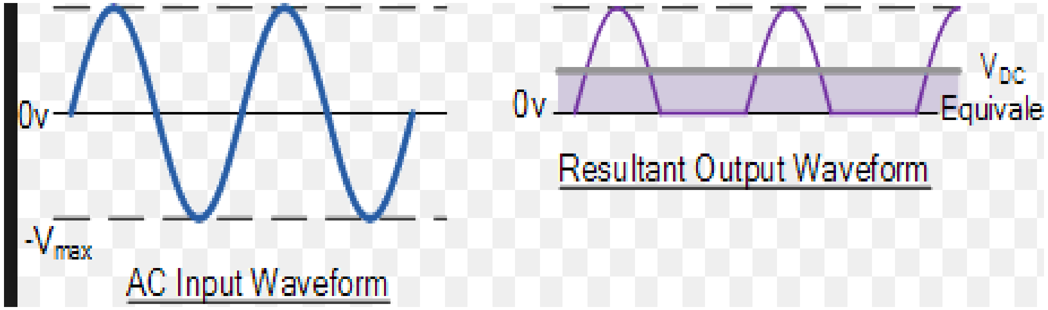
$= \frac{\pi}{2} I_m^2$

R.M.S. value $I_{rms} = \sqrt{\frac{\text{Area of first half of squared wave}}{\text{Base}}}$

$= \sqrt{\frac{\pi I_m^2}{2\pi}} = \sqrt{\frac{I_m^2}{2}} = 0.707 I_m$



R.M.S. Value of Half wave rectifier



$$\therefore \quad \text{rms current, } I = \sqrt{\frac{1}{2\pi}\int_0^{\pi} i^2\, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m{}^2 \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{I_m{}^2}{4\pi}\int_0^{\pi}(1 - \cos 2\theta)\, d\theta} = \sqrt{\frac{I_m{}^2}{4\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}}$$

$$= \sqrt{\frac{I_m{}^2}{4\pi}(\pi)} = \frac{I_m}{2} = 0.5\, I_m$$

$\therefore \quad I = 0.5\ I_m$ for a half wave rectified a.c.

Peak Factor

- Ratio of the Maximum value to the R.M.S. value of an alternating quantity

  It is denoted by

$$K = \frac{Maximum\,Value}{RMS\,Value}$$

Where K = 1.414

- Peak factor is also called the crest factor or amplitude factor.

Form Factor

Form Factor is ratio of R.M.S. value to the average value of an alternating quantity. It is denoted by K_f

$$K_f = \frac{R.M.S.\ Value}{Average\ Value}$$

Where K_f = 1.11

Question For Practice

Q: An alternating voltage is given by V= 282.8 sin 377t.

Find

(i) Frequency

(ii) RMS value

(iii) Average value

(iv) instantaneous value of voltage when t is 3 m sec.

A.C. Circuit

- Circuits in which currents and voltages vary sinusoidally, ie vary with time are called A.C. circuits.

  - All A.C. circuits are made up of combination of resistance R , inductance L and capacitance C.
  - The circuit elements R,L and C are called circuit parameters.

- To study a general A.C. circuit it is necessary to consider the effect of each separately.

Purely Resistive Circuit

- In purely resistive circuits, all the power is dissipated by resistors.
  - Voltage and current in same phase.

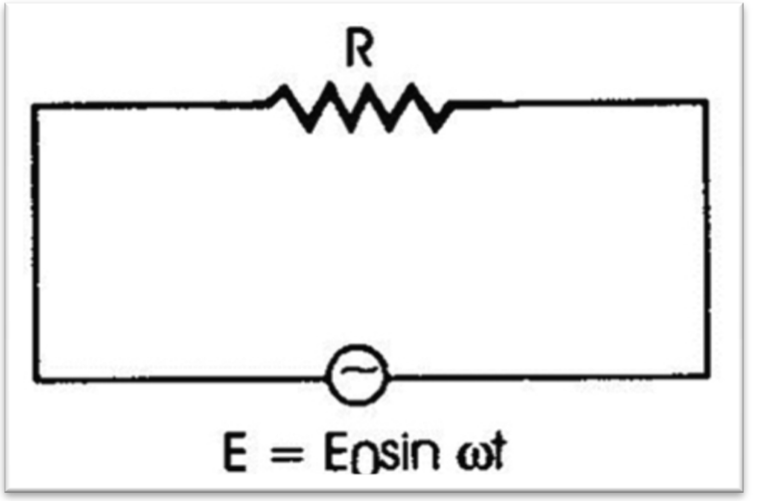
$V = V_m \sin \omega t$ ..... (1)

$I = \frac{V}{R} = V_m \frac{\sin\omega t}{R}$ (Using Ohm's Law)

At $\omega t = 90^\circ$, $\sin\omega t = 1$ & current will be maximum.

$I_m = \frac{V_m}{R}$

$I = I_m \sin\omega t$ ...... (2)



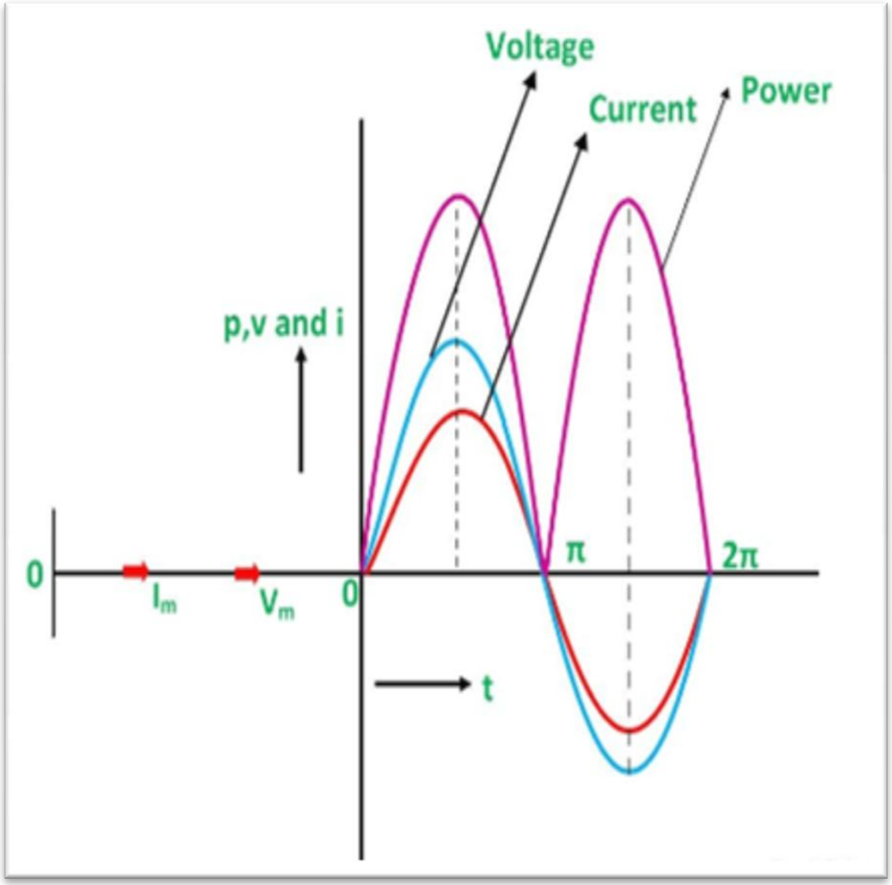
For purely resistive circuits, the voltage and current are in phase with each other .

Phasor and Wave Diagram of Purely Resistive Circuit

Average power consumed over a complete cycle

$P = V_{rms} I_{rms}$

$P = VI$



Purely Inductive Circuit

- In purely inductive circuits, current lags the voltage by an angle of 90 °
  - No power consumed in pure inductive circuit

The alternating voltage is

$V = V_m \, Sin\omega t$

An alternating current i flows through the inductance which induces an emf in it, given by

$e = -L \, di/dt$

This induced emf is equal and opposite of the applied voltage

$V = -e = -(-L \, di/dt)$

$V = L \, di/dt$

$V_m \, Sin\omega t = L \, di/dt$

$$di = \frac{Vm \, Sin\omega t}{L} \, dt$$

$$\int di = \int \frac{Vm \, Sin\omega t}{L} \, dt$$

$$i = \frac{Vm}{\omega L} \, (-cos\omega t)$$

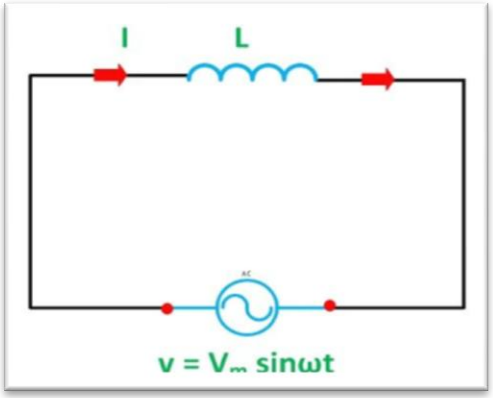
$$i = \frac{Vm}{\omega L} \, Sin\,(\omega t - 90°)$$

$$i = \frac{Vm}{XL} \, Sin\,(\omega t - 90°)$$

when $Sin\,(\omega t-90°) = 1$, value of current is maximum

$$I_m = \frac{Vm}{XL}$$

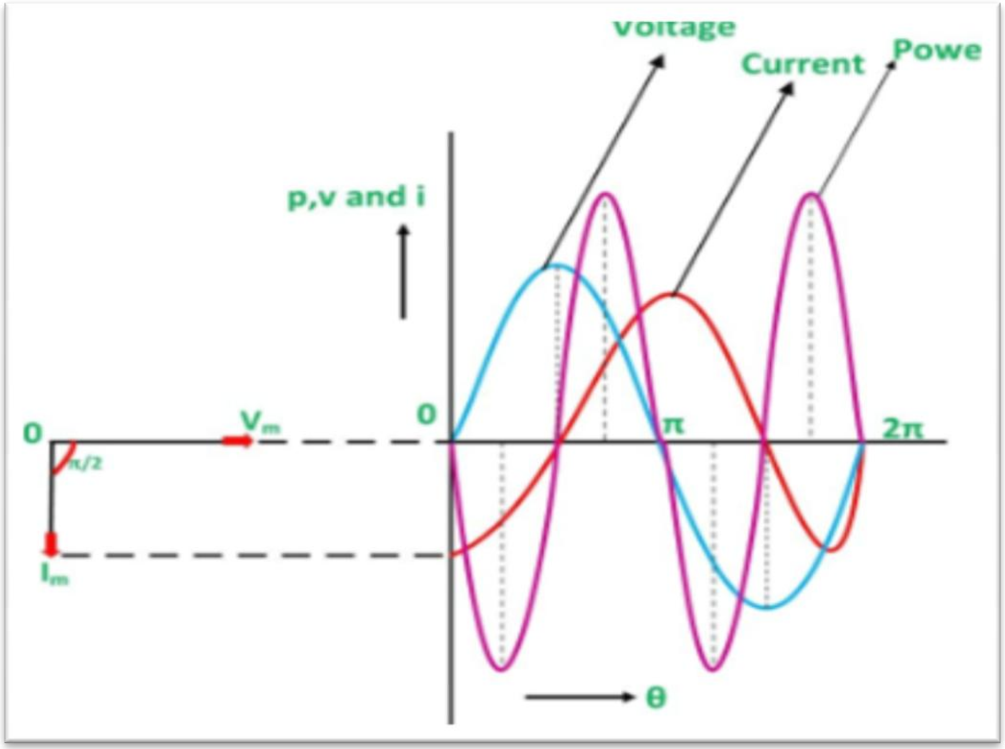
Therefore $i = I_m \, Sin\,(\omega t - 90°)$



Phasor and Wave Diagram of Purely Inductive Circuit

Average power consumed over a complete cycle

P= zero



Purely Capacitive Circuit

- In purely capacitive circuits, current leads the voltage by an angle of 90 °
  - No power consumed in pure capactive circuit

$V = V_m \sin \omega t$ .....(1)

The current in the circuit at any instant is

$i = dq/dt$

$i = d(CV)/dt = C\, dV/dt = C\, d(V_m \sin \omega t)/dt$

$i = \omega C V_m \cos \omega t = \omega C V_m \sin(\omega t + 90°)$

$i = \frac{V_m}{1/\omega C} \text{Sin}(\omega t + 90°)$

$i = \frac{V_m}{X_C} \text{Sin}(\omega t + 90°)$.......... (2)

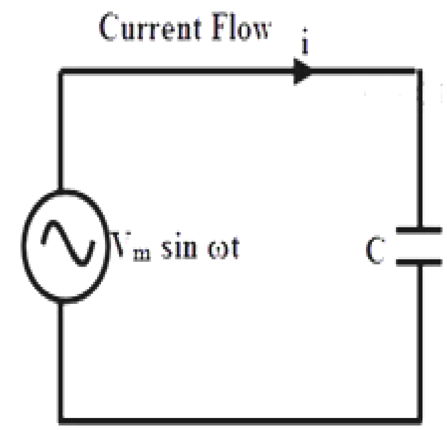
At Sin $(\omega t + 90°) = 1$, value of current will be maximum

i.e. $I_m = \frac{V_m}{X_C}$, substituting in eqn(2)

$i = I_m \text{Sin}(\omega t + 90°)$.......... (3)

By comparing eqn (1) and (3),

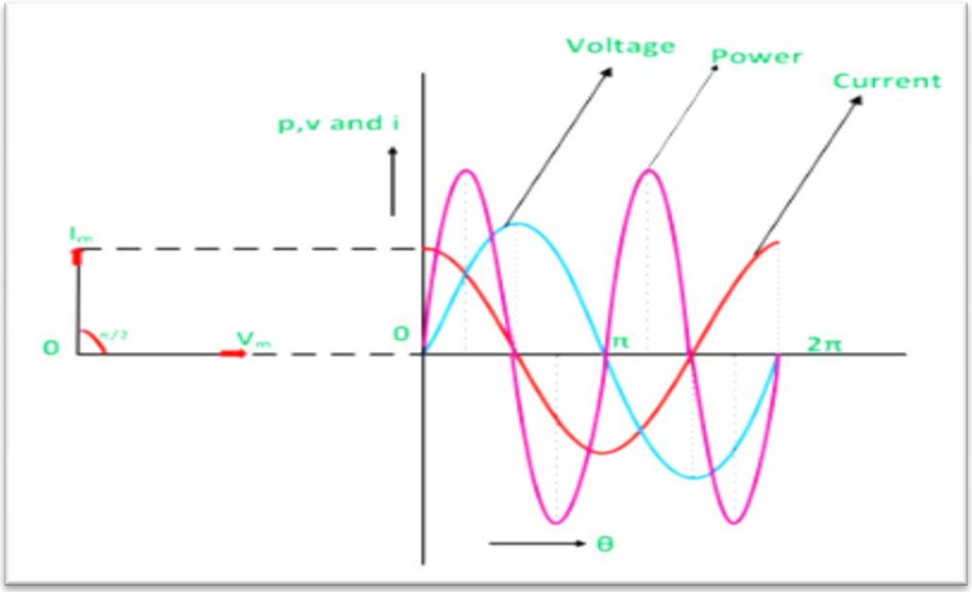
Current leads the voltage by an angle of 90 °



Phasor and Wave Diagram of Purely Capacitive Circuit

Average power consumed over a complete cycle

P= zero



A.C. Series Circuit

- In actual practice, A.C. circuit contain two or more than two components (R,L,C) connected in series or parallel.

  – Three types of A.C. series circuits
    - R-L series circuit
    - R-C series circuit
    - R-L-C series circuit

R-L Series Circuit

This circuit contains a resistance R and an inductance L in series

Let V = supply voltage

I = circuit current

V_R= voltage drop across R = IR

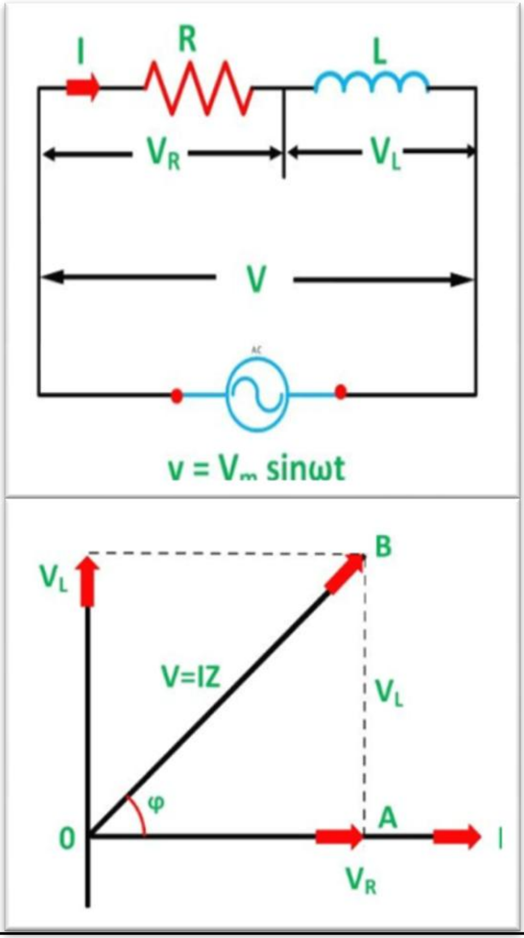
V_L = voltage drop across L = IX_L= 2πfLI

Φ_L = phase angle between I and V

Since I is common to both elements R and L, this is used as reference phasor.

The voltage V_R is in phase with I and V_L leads I by 90 ° .

The voltage V is the phasor sum of V_R and V_L

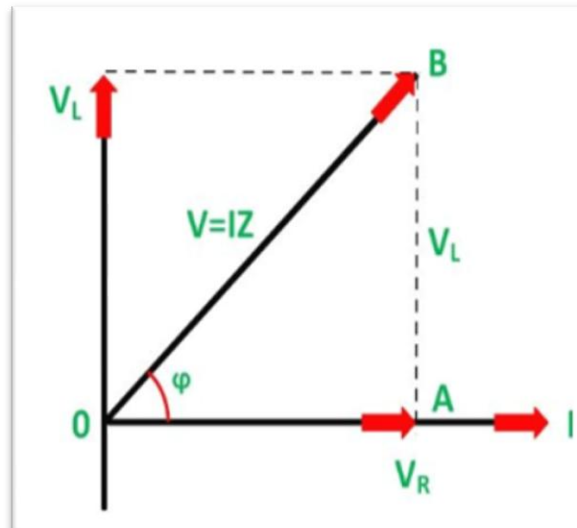


Phasor Diagram of R-L Series Circuit

The triangle having V_R,V_L and V as its sides is called voltage triangle for a series R-L circuit.

The phase angle Φ_L between the supply voltage V and the circuit current I is the angle between the hypotenuse and the side V_R .

It is seen that the current I is lagging behind the voltage V in an R-L circuit.



$V^2 = V^2_R + V^2_L$

$V^2 = (RI)^2 + (X_L I)^2$

$V^2/I^2 = R^2 + X^2_L$

$V/I = \sqrt{(R^2 + X^2_L)}$

$Z_L = \sqrt{(R^2 + X^2_L)}$

Z_L is called the impedance of a series R-L circuit

$Z_L = V/I$

$V = Z_L I$

Impedance Triangle For R-L Series Circuit

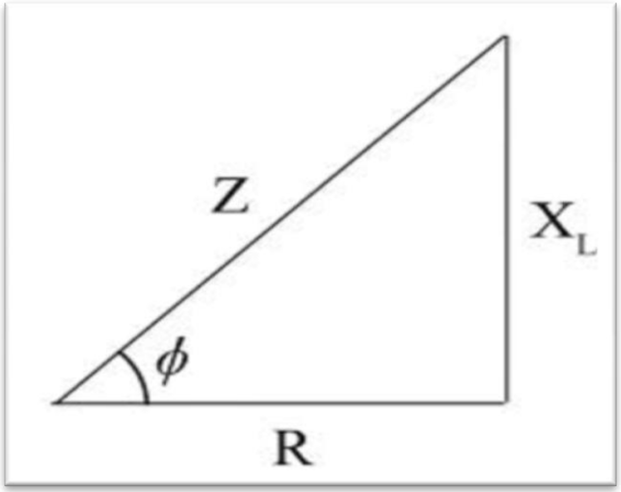
- If the length of each side of the voltage triangle is divided by current I, the impedance triangle is obtained .
  - The following results may be found from an impedance triangle for a series R-L circuit:

$Z_L = \sqrt{(R^2 + X_L^2)}$

$R = Z_L \cos\Phi_L$

$X_L = Z_L \sin\Phi_L$

$\tan\Phi_L = X_L / R$

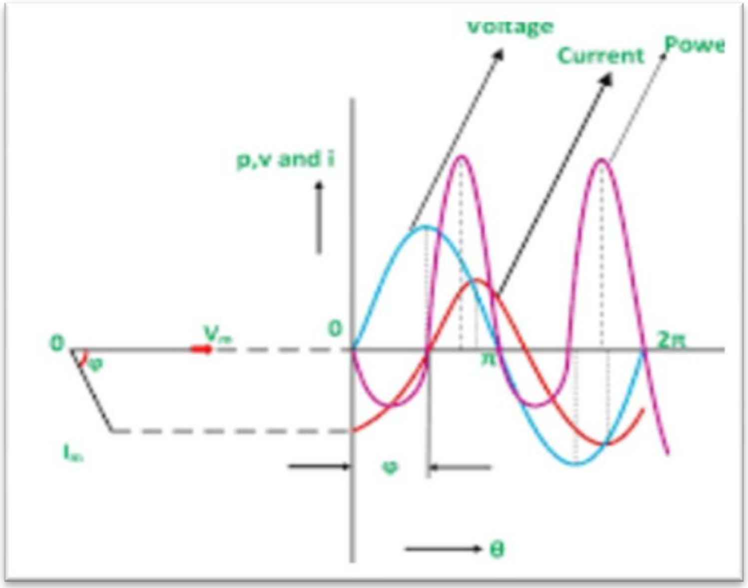


Phasor and Wave Diagram of R-L Series Circuit

Average power consumed over a complete cycle

$P = V_{rms} I_{rms} \cos\phi$

$P = V I \cos\phi$



R-C Series Circuit

This circuit contains a resistance R and a capacitance C in series

Let V = supply voltage

I = circuit current

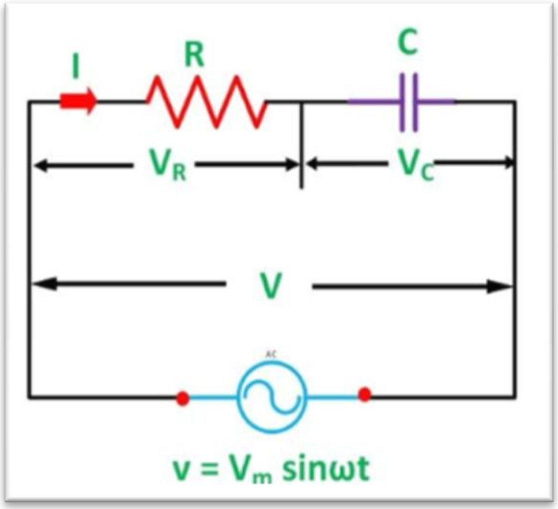
V_R = voltage drop across R = IR

V_C = voltage drop across C =IX_C= I/2πfC

Φ_C = phase angle between I and V

The voltage V_R is in phase with I and V_C lags I by 90° .

The voltage sum is $V = V_R + V_C$



Phasor Diagram of R-C Series Circuit

The triangle having V_R,V_C and V as its sides is called voltage triangle for a series R-C circuit.

The phase angle Φ_C between the supply voltage V and the circuit current I is the angle between the hypotenuse and the side V_R .

It is seen that the current I is leading behind the voltage V in an R-C circuit.

$V^2 = V^2_R + V^2_C$

$V^2 = (RI)^2 + (X_C I)^2$

$V^2/I^2 = R^2 + X^2_C$

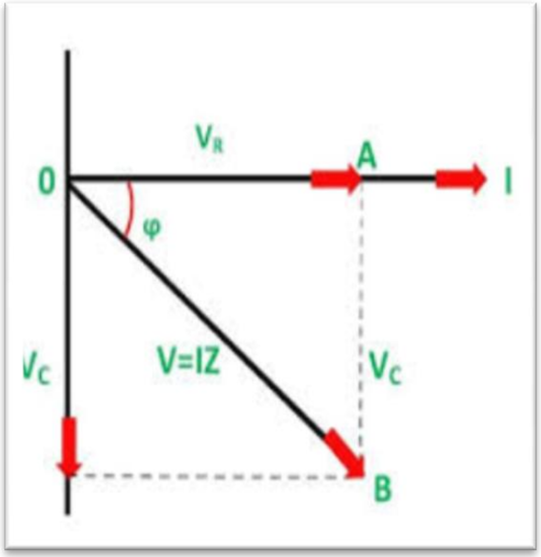
$V/I = \sqrt{(R^2 + X^2_C)}$

$Z_C = \sqrt{(R^2 + X^2_C)}$

Z_C is called the impedance of a series R-C circuit

$Z_C = V/I$

$V = Z_C I$



Impedance Triangle For R-C Series Circuit

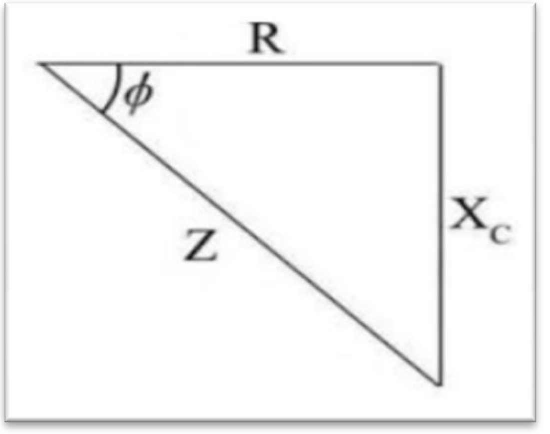
- If the length of each side of the voltage triangle is divided by current I , the impedance triangle is obtained .
  - The following results may be found from an impedance triangle for a series R-C circuit.

$Z_c = \sqrt{(R^2 + X^2_c)}$

$R = Z_c \cos\Phi_c$

$X_c = Z_c \sin\Phi_c$

$\tan\Phi_c = X_c / R$

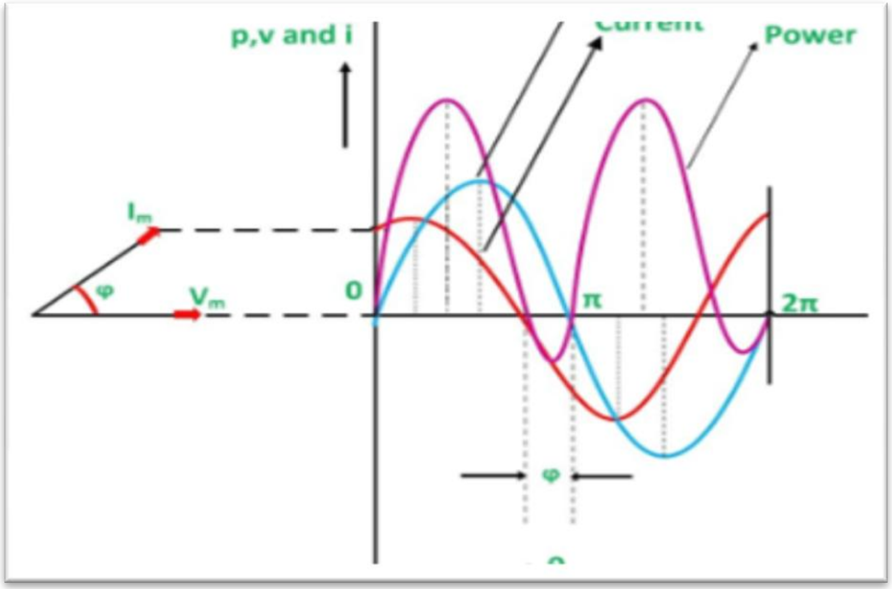


Phasor and Wave Diagram of R-C Series Circuit

Average power consumed over a complete cycle

$P = V_{rms} I_{rms} \cos\phi$

$P = V I \cos\phi$



R-L-C Series Circuit

- A circuit having R, L and C in series is called a R-L-C series circuit

.

- Current is used as reference phasor in series circuit since it is common to all the elements of circuit.

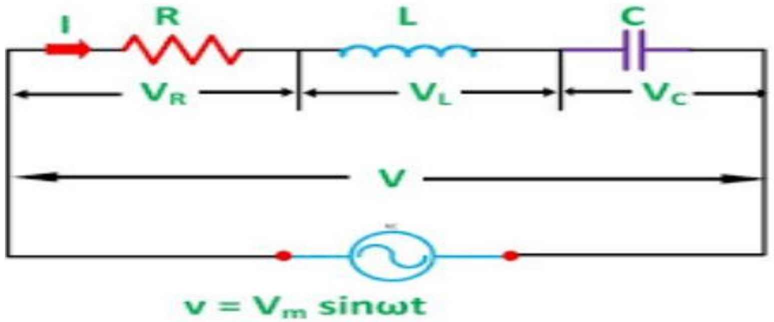
- There are four voltages

V_R in phase with I

V_L leading I by 90^0

V_C lagging I by 90^0

Total voltage $V = V_R + V_L + V_C$



Phasor Diagram of R-L-C Series Circuit

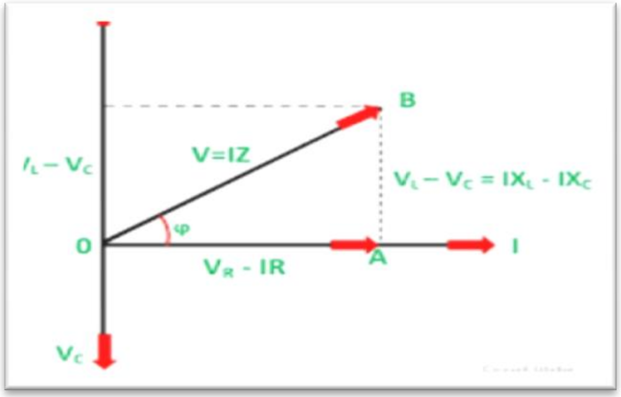
- V_L and V_C are in opp. directions and their resultant is their arithmetic difference.

  - There are three possible cases in series RLC circuit

    $a)\ V_L < V_c$ i.e. $X_L < X_c$

    $b)\ V_L > V_c$ i.e. $X_L > X_c$

    $c)\ V_L = V_c$ i.e. $X_L = X_c$



When $X_L > X_C$, the circuit is predominantly Inductive .

Inductive circuits cause the current 'lag' the voltage.

$$V = I\sqrt{[R^2 + (X_L - X_C)^2]}$$

$$Z = \sqrt{[R^2 + (X_L - X_C)^2]}$$

When XL < XC the circuit is predominately Capacitive.

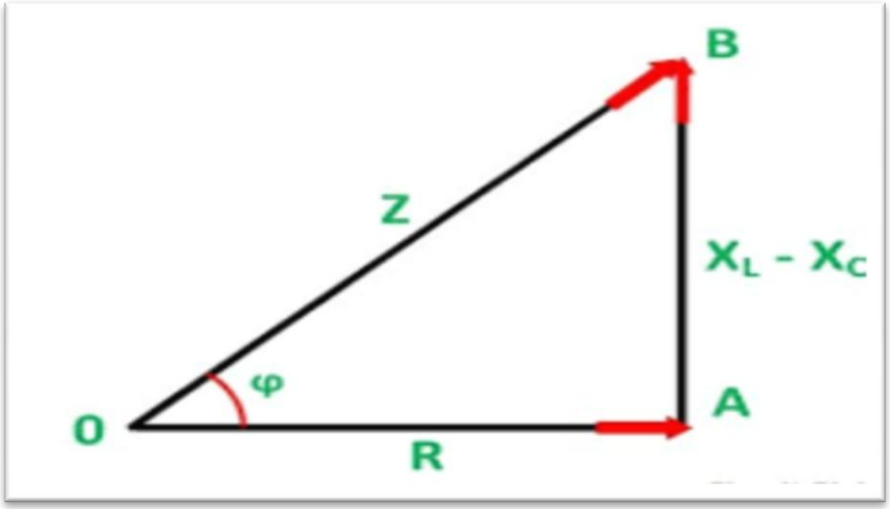
Capacitive circuits cause the current to 'lead' the voltage.

$$V = I\sqrt{[R^2 + (X_c - X_L)^2]}$$

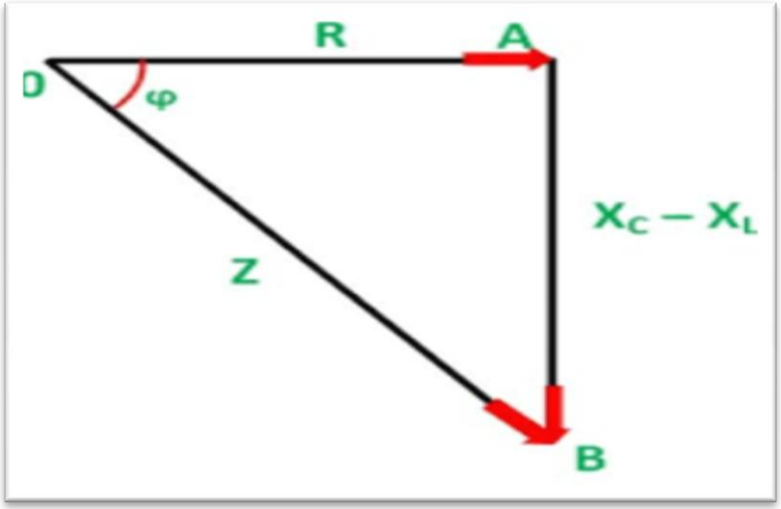
$$Z = \sqrt{[R^2 + (X_c - X_L)^2]}$$

Impedance Triangle For R-L-C Series Circuit

If the length of each side of a voltage triangle is divided by current I, the impedance triangle is obtained. The impedence triangle for series R-L-C circuit.



$X_L > X_C$



$X_C > X_L$

Question For Practice

Q: A coil resistance 10 Ω and inductance 114.7mH is connected in series with capacitor of 159.16μF across a 200V, 50 Hz supply.

Calculate

(i) Inductive reactance

(ii) Capacitive reactance

(iii) Impedance

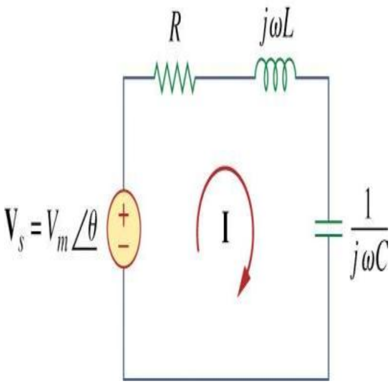
(iv) Current

(v) Voltage across coil and capacitor

Resonance

- Resonance is a condition in an RLC circuit in which the capacitive and inductive reactance are equal in magnitude, thereby resulting in a purely resistive impedance.

  - At resonance, the impedance consists only resistive component R.
  - The value of current will be maximum since the total impedance is minimum.
  - The voltage and current are in phase.
  - Maximum power occurs at resonance since the power factor is unity
  - Resonance circuits are useful for constructing filters and used in many application.



Resonance Frequency

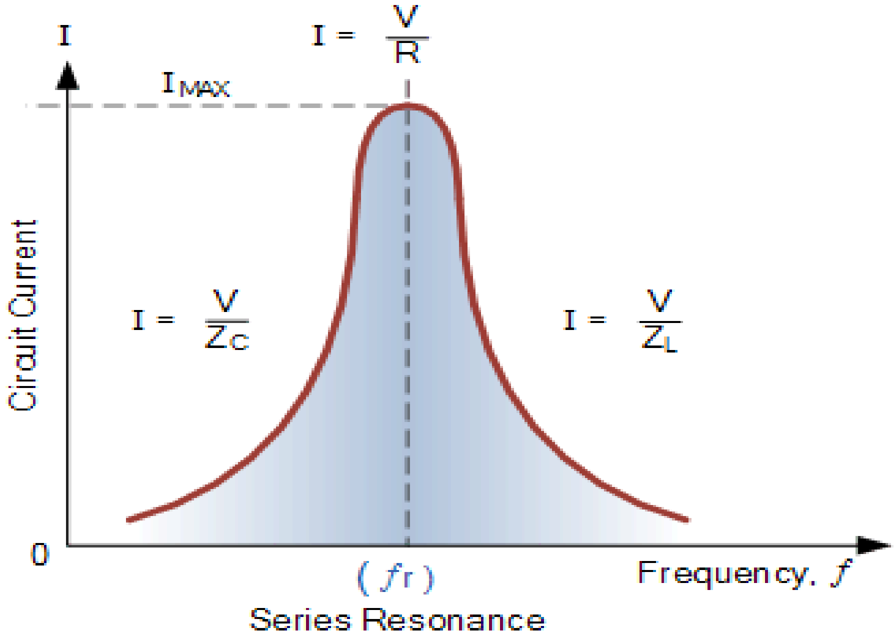
- Resonance frequency is the frequency where the condition of resonance occur.
  - Also known as center frequency.

$$\omega_o = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

Resonance Curve

- The curve obtained by plotting a graph between current and frequency is known as resonance curve or response curve .
- The current has a maximum value at resonance given by I_{max} = V/R.
- The value of I decreases on either sides of the resonance



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